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Linear viscosity and elasticity periodic regularity problem solution

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ABSTRACT

This paper proposes a new method for solution of integral-differential equation of oscillation of linear viscous and elastic systems using the Laplace integral transform method for random hereditary functions with low viscosity of materials, and it was demonstrated that this solution is more accurate in case of large values of oscillation frequency, as compared to those present in the literature. A solution of the integral-differential equation of oscillation of linear viscoelastic systems was developed in expanded form, and it was demontrated that the alpha of that expanded form is a relevant solution of that equation which was obtained by a well-known method of averaging, and orifginal functions of the first two terms of the expanded form. The influence of the second term of the expanded form on the solution for Rizhanin weakly-singular kernel with the parameters for materials of KAST-V fiberglass. A fundamental result was achieved, that in the case of low frequency, the influence of subsequent terms of the extended form on the solution is insignificant, and that is growing with the rise of the oscillation frequency. When the frequency rate reaches one hundred the amplitude of the second term of the extended form, at some given points of time, is 20-25% of the amplitude of the first term, and herewith, the amplitudes of all terms of the extended form with the course of time are distributed according to the exponential law, and the phases are shifted.

1. Introduction

A wide-range use of polymer, composite, and other materials with rheological properties in technology, machine-building, and in various industries, in the form of plates and envelopes, has led to the necessity of study of the problems of optimum structural design. The mentioned thin-walled elements are used in contemporary constructions as both buildings' envelopes and supporting structures deigned for operation under the effect of power load. It is clear that calculations of strength, stability and oscillations of the described structures have a crucial importance during the constructions designing process.

In such a way, in order to obtain an actual picture of the tensed deformation state of structural elements, it is necessary to conduct the research with taking into consideration the rheological properties of materials subjected to intense dynamic loads.

Because of that, the heritable theory of viscoelasticity attracts the attention of researchers. In connection with that, in recent years, a number of scientific papers were published, which represent the most recent achievements in the theory of viscoelasticity.

Despite the above mentioned studies in the area in question, until now there are no methods in place which would, to a sufficient enough extent, analyze the tension deformation state of a viscoelastic object.

Analyzing the problem of propagation of non-steady dynamic waves in linear viscoelastic materials, two quite different problems are encountered. The first issue lies in the description of differential equation of motion, selection of equations of state, and establishing initial and boundary conditions. And the second issue, usually the more complicated one, is to solve the problem. Among dynamic viscoelasticity problems, the problems of oscillations of viscoelastic systems and non-steady dynamic waves' problems, special mention has to be given to problems of viscoelastic systems' oscillation and non-steady wave problems, which are topical and have practical significance.

2. Methods of the study

For analytical solution of the set problem, the following methods were used: mathematical methods of the dynamic theory of linear and non-linear viscoelasticity, the theory of non-linear differential equations in partials derivatives of

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hyperbolic type, the theory of Volterra integral equations of the second type, operational calculus, and the method of variables' separation.

Using these methods, the following was developed: a generalized mathematical model, and a method for solving the problem of free motion of heritable solid objects, with the use of which it will be possible to describe the behavior of viscoelastic systems with random rheology under the effect of external load.

3. Topicality of the study

The advancement of different areas of technology and industry is inseparably associated with development of new polymer and composite materials with preset physical and mechanical properties.

The emergence of such materials will become widely used in the emerging technology and contributes to development of new structures operating in non-standard conditions, such as accidental non-steady effects, pressure of motional load, impact of shockwaves, and seismic effects.

Such practical problemss of the theory of linear and non-linear viscoelasticity remains, as of today, understudied.

Because they are not just topical, but also important problems of mechanics of a deformed solid object for practical application.

These circumstances predetermine the topicality of the subject of this paper, aimed at development and analysis of mathematical models enabling the development of efficient methods for solution of the set problems.

4. Review of literature

The study of physical-mechanical properties of materials with rheological properties, and the analysis of application thereof in industry and technology, etc., demonstrated the necessity of use the methods of dynamic viscoelasticity theory (Lebedev, 1982) for valuation of structures' stability and strength.

Among dynamic problems of viscoelasticity, the problems connected with oscillation of viscoelastic systems and non-steady wave problems have to be specially mentioned.

Especially complex are non-steady dynamic problems of viscoelasticity which have important practical applications in many areas of contemporary technology. In order to solve noon-steady dynamic problems of viscoelasticity, there are various mathematical methods which allow to obtain solutions of boundary problems of propagation of non-steady waves (Badalov, 1987; Bazhlekova, 2014) and the problems of oscillation of viscoelastic systems for certain heritable functions (Matiash, 1971). In papers (Badalov, 1987; Mainardi, 2012; Bazhlekova, 2014), non-linear oscillations of viscoelastic system using the method of double integration in time were studied, and for numerical solution, the method of direct replacement of integrals by a bounded sum.

When solving the problems of oscillations of viscoelastic systems presented in papers (Matiash, 1971; Badalov, 1987), a well-known averaging method was used, which found its further development in scientific papers (Lebedev, 1982; Zhao, 2009).

It should be noted that in many cases, when solving the problem of non-steady dynamic problems of viscoelasticity, analytical form of relaxation kernels is not prescribed. Herewith, the solutions are framed with the use of certain approximation methods which deduce the final solution to the solution of integral-differential equations of free and forced oscillations of viscoelastic systems implemented using different numerical methods (Badalov, 1987; Mainardi, 2012; Bazhlekova, 2014). Especially complex and important task is to conduct an in-depth analysis and developed more accurate solutions of the integral-differential equation of oscillation of viscoelastic systems, and to study, using thereof, the influence of different boundary and initial conditions, rheological properties and inhomogeneity of materials, etc., on wave field (Matiash, 1971; Mainardi, 2012; Kurbanov, 2014). This paper is dedicated to study of these problems.

5. Results of the study

The influence of rheological properties of materials on oscillations of thin-walled shells is studied in this paper.

Let us consider the problem of free oscillation of circular cylindrical shell. Let us choose the coordinate $x_1=x$ in the middle surface of the shell along the generator as coordinates and arc length $x_2 = \varphi$ in the circular direction. Herewith, the equations of the fluctuation of elastic circular cylindrical shell in motions are of the form:

$$L_k[U_1, U_2, U_3] = \rho h \frac{\partial^2 U_k}{\partial t^2} \qquad \left(k = \overline{1,3}\right)$$

(1)

Where ρ is the density of the material?

h is the thickness of the shell;

U_k is the motion.

If the material of the shell is deemed viscoelastic, the relation between the tension σ and deformation ε is of the form (Kurbanov, 2014):

$$S_{ij}(t) = 2G\left[\varepsilon_{ij}(t) - \int_0^t Q(t-\tau)\varepsilon_{ij}(\tau)\,d\tau\right]$$
$$\sigma(t) = k\left[\theta(t) - \int_0^t Q_1(t-\tau)\theta(\tau)\,d\tau\right] \qquad (2)$$

Where

$$Q(t) = -\frac{1}{2G}R'(t),$$
$$Q_1(t) = -\frac{1}{k}R_1'(t) \qquad (3)$$

R(t) and *R$_i$(t)* are called functions of shift and three-dimensional relaxation accordingly;

G is instantaneous shift modulus;

K is instantaneous modulus of three-dimensional compression.

Let us rewrite expressions (2) in the form:

$$\sigma_{ij}(t)=s_{ij}+\sigma=2G\left[\varepsilon_{ij}-\int_0^t Q(t-\tau)\varepsilon_{ij}(\tau)d\tau\right]+ +\frac{3k-2G}{3}\left[\theta(t)-\int_0^t Q(t-\tau)\theta(\tau)d\tau\right]\delta_{ij}+ +k\delta_{ij}\int_0^t\left[Q(t-\tau)-Q_1(t-\tau)\right]\theta(\tau)d\tau \quad (4)$$

Let

$$\int_0^t\left[Q(t-\tau)-Q_1(t-\tau)\right]\theta(\tau)d\tau=0,$$

Then

$$\sigma_{ij}=\frac{E}{1+\nu}E^*(\varepsilon_{ij})+\frac{E}{(1+\nu)(1-2\nu)}E^*(\theta)\delta_{ij} \quad (5)$$

Where E is Young's modulus;

ν is the Poisson's ratio;

And the operator $E^*(z)$ is designated by the formula:

$$E^*(z)=z-\int_0^t Q(t-\tau)z(\tau)d\tau \quad (6)$$

We assume that the Kirchhoff-Love hypothesis is fulfilled, and the isotropy of the material is preserved. Then, when $\sigma_{33}=0$

$$E^*(\varepsilon_{33})=-\frac{\nu}{1-\nu}E^*(\varepsilon_{11}+\varepsilon_{22}) \quad (7)$$

And equation (5) obtains the form:

$$\sigma_{11}=\frac{E}{1-\nu^2}\left[E^*(\varepsilon_{11})+\nu E^*(\varepsilon_{22})\right]$$

$$\sigma_{22}=\frac{E}{1-\nu^2}\left[E^*(\varepsilon_{22})+\nu E^*(\varepsilon_{11})\right],$$

$$\sigma_{12}=\frac{E}{1+\nu}E^*(\varepsilon_{12})$$

Taking into account these equivalences in the equations of moment coefficients and cutting forcers, we obtain:

$$T_1=\frac{Eh}{1-\nu^2}\left[E^*(\varepsilon_1)+\nu E^*(\varepsilon_2)\right],$$

$$T_2=\frac{Eh}{1-\nu^2}\left[E^*(\varepsilon_2)+\nu E^*(\varepsilon_1)\right]$$

$$T_{12}=\frac{E}{2(1+\nu)}E^*(\omega),$$

$$M_1=\frac{Eh^3}{12(1-\nu^2)}\left[E^*(\aleph_1)+\nu E^*(\aleph_2)\right]$$

$$M_2=\frac{Eh^3}{12(1-\nu^2)}\left[E^*(\aleph_2)+\nu E^*(\aleph_1)\right], \quad (8)$$

$$M_{21}=M_{12}=\frac{Eh^3}{12(1+\nu)}E^*(\tau)$$

These indicate that from relevant equations of elastic shell oscillation, it is quite simple to obtain relevant equation of viscoelastic shell oscillation, if parameters $\{\varepsilon_1,\varepsilon_2,\omega,\aleph_1,\aleph_2,\tau\}$ are replaced with operators $\{E^*(\varepsilon_1), E^*(\varepsilon_2), E^*(\omega), E^*(\aleph_1), E^*(\aleph_2), E^*(\tau)\}$ determined using the formula (6). Because of that, from equation (2) for viscoelastic shell, the following formula is obtained (Matiash, 1971; Zhao, 2009):

$$L_k\left[E^*(U_1), E^*(U_2), E^*(U_3)\right]=\rho h\frac{\partial^2 U_k}{\partial t^2} \quad (9)$$

It should be mentioned that initial and boundary conditions have to be attached to equation (9) describing the oscillations of viscoelastic shell.

Boundaries conditions may be set in different ways and are used for the determination of own numbers and own functions, made in the first section of this chapter, and the initial conditions are set in the following form:

$$U\big|_{t=0}=T(t)\big|_{t=0}=T_0;$$

$$\left.\frac{\partial U}{\partial t}\right|_{t=0}=T'(t)\big|_{t=0}=T_1' \quad (10)$$

In many practical problems, the study of viscoelastic systems oscillation is reduced to solution of integral-differential equations which are obtained from equation (9), or using thee variables' separation method, or using the Bubnov-Galerkin method. After doing that, the following is obtained from equation (9):

$$T''(t)+\lambda^2 T(t)=\varepsilon\lambda^2\int_0^t\omega(t-\tau)T(\tau)d\tau, \quad (11)$$

where

$$Q(t)=\varepsilon\omega(t),$$

$$\varepsilon=Q(t_0),$$

$$0\le\omega(t)\le1$$

From that it is seen that the solution of the set problem (Matiash, 1971) is reduced to the solution of equation (11) using conditions (10).

Applying the Laplace integral transform in time t to equation (11), the following is obtained:

$$\overline{T}(p)=\frac{pT_0+T_0'}{p^2+\lambda^2-\varepsilon\lambda^2\varpi(p)} \qquad (12)$$

It is clear that in case of small values of time t, the parameter p is large, and with the increase of p, the image of relaxation kernel $\varpi(p)$ tends to zero. Because of that, the in equation

$$0\le\varepsilon\int_0^t\omega(t)d\tau<<1\,,\quad \varepsilon\,\omega(t)\ge 0$$

Established by A. A. Ilyushin (1959) is met here. It follows from that in equation that the condition

$$\left|\frac{\varepsilon\,\lambda^2\,\varpi(p)}{p^2+\lambda^2}\right|<1$$

is always correct.

With such presumptions, the right side of the equivalence (12) is expanded into series:

$$\overline{T}(p)=\frac{pT_0+T_0'}{p^2+\lambda^2}\sum_{n=0}^{\infty}\left(\frac{\varepsilon\,\lambda^2\,\varpi(p)}{p^2+\lambda^2}\right)^n \qquad (13)$$

Let us make the following transformations.

$$L^{-1}\left[\frac{\varepsilon\lambda^2\overline{\omega}(p)}{p^2+\lambda^2}\right]=\varepsilon\lambda\int_0^t\omega(\tau)\sin\lambda(t-\tau)=\varepsilon\lambda\sin\lambda t\int_0^\infty\omega(\tau)\cos\lambda\tau d\tau-\varepsilon\lambda\cos\lambda t\int_0^\infty\omega(\tau)\sin\lambda\tau d\tau-$$

$$-\varepsilon\lambda\sin\lambda t\int_t^\infty\omega(\tau)\cos\lambda\tau d\tau+\varepsilon\lambda\cos\lambda t\int_t^\infty\omega(\tau)\sin\lambda\tau d\tau$$

Let us introduce the following notations:

$$\omega_s=\int_0^\infty\omega(\tau)\sin\lambda\tau d\tau\;;$$

$$\omega_c=\int_0^\infty\omega(\tau)\cos\lambda\tau d\tau$$

$$g(t)=\sin\lambda t\int_t^\infty\omega(\tau)\cos\lambda\tau d\tau-\cos\lambda t\int_t^\infty\omega(\tau)\sin\lambda\tau d\tau$$

Then, the last formula can be written in the form:

$$\frac{\varepsilon\lambda^2\overline{\omega}(p)}{p^2+\lambda^2}=\frac{\varepsilon\lambda\left[\omega_c-p\omega_s-\left(p^2+\lambda^2\right)\overline{g}(p)\right]}{p^2+\lambda^2}$$

Taking into consideration this expression for $\overline{T}(p)$ the following is obtained:

$$\overline{\mathrm{T}}(p)=\frac{T_0p+T_0^1}{\overline{\varphi}(p)-\varepsilon\lambda^2\overline{\psi}(p)} \qquad (14)$$

Here

$$\overline{\varphi}(p)=\left(p+\frac{1}{2}\varepsilon\omega_s\lambda\right)^2+\lambda^2\left(1-\frac{1}{2}\varepsilon\omega_c\right)^2$$

$$\overline{\psi}(p)=\overline{\omega}(p)+\omega_s\frac{p}{\lambda}+\omega_c+\frac{\varepsilon}{4}\left(\omega_s^2+\omega_c^2\right)$$

If the expression

$$g(t)=\int_t^\infty\omega(\tau)\sin\lambda(t-\tau)d\tau$$

is a small quantity, then $\left|\varepsilon\lambda^2\overline{\psi}(p)\right|$ will be small at $t\to\infty$.

Due to that, the following in equation is correct:

$$\left|\frac{\varepsilon\lambda^2\overline{\psi}(p)}{\overline{\varphi}(p)}\right|<0$$

Herewith, the following form can be written:

$$\overline{T}(p)=\frac{pT_0+T_0^1}{\overline{\varphi}(p)}\left[1+\varepsilon\lambda^2\frac{\overline{\psi}(p)}{\overline{\varphi}(p)}+\varepsilon^2\lambda^4\left(\frac{\overline{\psi}(p)}{\overline{\varphi}(p)}\right)^2+\dots\right] \qquad (15)$$

The original of the first term of this series is determined by the formula

$$T_1(t)=e^{-\frac{1}{2}\varepsilon\omega_s\lambda t}\left[T_0\cos\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t+\frac{T_0^1-\frac{1}{2}\varepsilon\lambda\omega_s}{\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)}\sin\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t\right] \qquad (16)$$

This is a known solution of the set problem obtained using the method of averaging (Matiash, 1971).

The original of the second term of the series is determined in the form:

$$T_2(t)=\varepsilon\lambda^2T_1(t)*Z(t) \qquad (17)$$

where

$$Z(t) \doteq \frac{\overline{\psi}(p)}{\overline{\varphi}(p)}$$

The original of the ratio $\frac{\overline{\psi}(p)}{\overline{\varphi}(p)}$ is determined by the formula:

$$Z(t) = \omega(t) * e^{-\frac{1}{2}\varepsilon\omega_s\lambda t} \frac{\sin\lambda\left(1-\frac{1}{2}\varepsilon\omega_s\right)t}{\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)} + \frac{\omega_s}{\lambda} e^{-\frac{1}{2}\varepsilon\omega_s\lambda t}\left[\cos\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t + \right.$$

$$\left. + \frac{d-\frac{\varepsilon}{2}\lambda\omega_s}{\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)}\sin\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t\right] \qquad (18)$$

where

$$d = \lambda\frac{\omega_c}{\omega_s} + \frac{\varepsilon\lambda}{4\omega_c}\left(\omega_s^2 + \omega_c^2\right)$$

The asterisk means the convolution of the functions

$$f(t) * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau$$

Taking into account (18) in (17), the original of the second term of the series is found.

In a similar way, it is possible to determine the originals of next terms of the solutions' series.

In order to compute the influence of the second term (17) on the solution $T_1(t)$, let's consider the Rzhanitsyn's kernel:

$$\omega(t) = \varepsilon t^{\alpha-1}e^{-\beta t} \qquad (19)$$

where $0 \le \alpha < 1$, β is a constant, ε is some small parameter.

For this kernel, the formula (16) does not change its form, and $T_2(t)$ is determined by the formula

$$T_2(t) = e^{-\frac{1}{2}\varepsilon\omega_s t}\left\{\left[\frac{\varepsilon m_1}{2}\cos\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t + \frac{\varepsilon m_4}{2}\sin\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t\right] \times \int_0^t e^{-\beta\tau}\tau^{\alpha-1}\sin 2\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)\tau d\tau + \right.$$

$$+\left[\frac{\varepsilon m_1}{2}\sin\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t - \frac{\varepsilon m_4}{2}\cos\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t\right] \times \int_0^t e^{-\beta\tau}\tau^{\alpha-1}d\tau -$$

$$-\left[\frac{\varepsilon m_1}{2}\sin\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t - \frac{\varepsilon m_4}{2}\cos\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t\right]\int_0^t e^{-\beta\tau}\tau^{\alpha-1}\cos 2\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)\tau d\tau +$$

$$\left. + \frac{1}{2}(m_2 - m_6)\cos\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t + \left[\frac{m_2 - m_6}{2\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)} + \frac{1}{2}(m_3 + m_5)\right]\sin\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t\right\} \qquad (20)$$

where

$$\omega_s = \frac{\varepsilon Q(\alpha)}{\left(\beta^2+\lambda^2\right)^{-\alpha/2}}\sin\left[\alpha\, arctg\left(\frac{\lambda}{\beta}\right)\right]$$

$$\omega_c = \frac{\varepsilon Q(\alpha)}{\left(\beta^2+\lambda^2\right)^{-\alpha/2}}\cos\left[\alpha\, arctg\left(\frac{\alpha}{\beta}\right)\right]$$

$$m_1 = \frac{\varepsilon\lambda T_0}{1-\frac{1}{2}\varepsilon\omega_c}; \qquad m_2 = \varepsilon\lambda T_0\omega_s$$

$$m_3 = \frac{\varepsilon\omega_s T_0\left(d - \frac{1}{2}\varepsilon\omega_s\lambda\right)}{1-\frac{1}{2}\varepsilon\omega_c}$$

$$m_4 = \frac{\varepsilon\left(T_0{}^1 - \frac{1}{2}\varepsilon\omega_s T_0\right)}{\left(1-\frac{1}{2}\varepsilon\omega_c\right)^2}$$

$$m_5 = \frac{\varepsilon\omega_s\left(T_0^{\,1} - \frac{1}{2}\varepsilon\lambda T_0\omega_s\right)}{1 - \frac{1}{2}\varepsilon\omega_c};$$

$$m_6 = \frac{\varepsilon\omega_s\left(T_0^{\,1} - \frac{1}{2}\varepsilon\omega_s T_0\lambda\right)\left(d - \frac{1}{2}\varepsilon\omega_s\lambda\right)}{\lambda\left(1 - \frac{1}{2}\varepsilon\omega_c\right)^2}$$

Q(α) is Euler's gamma function.

Integrals included to (20) for certain KAST-V fiberglass material were computed using approximation methods, and graphs of $T_1(t)$ and $T_2(t)$ were plotted as functions of time t with the following values of parameters $\alpha = 0{,}2$, $\beta = 0{,}004$; $\lambda = 100$, $T_0 = 0$, $T_0^1 = 1$; $T_0 = 1$, $T_0^1 = 1$;

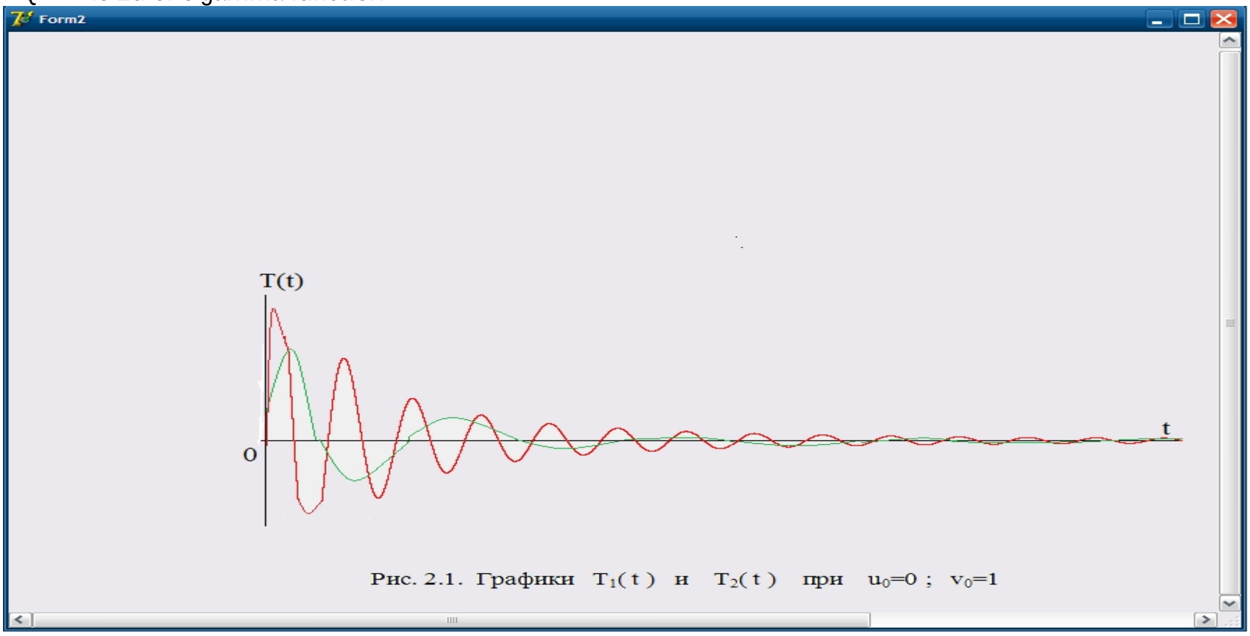

Fig. 1: Graphs of $T_1(t)$ and $T_2(t)$ at $u_0=0$; $v_0=1$

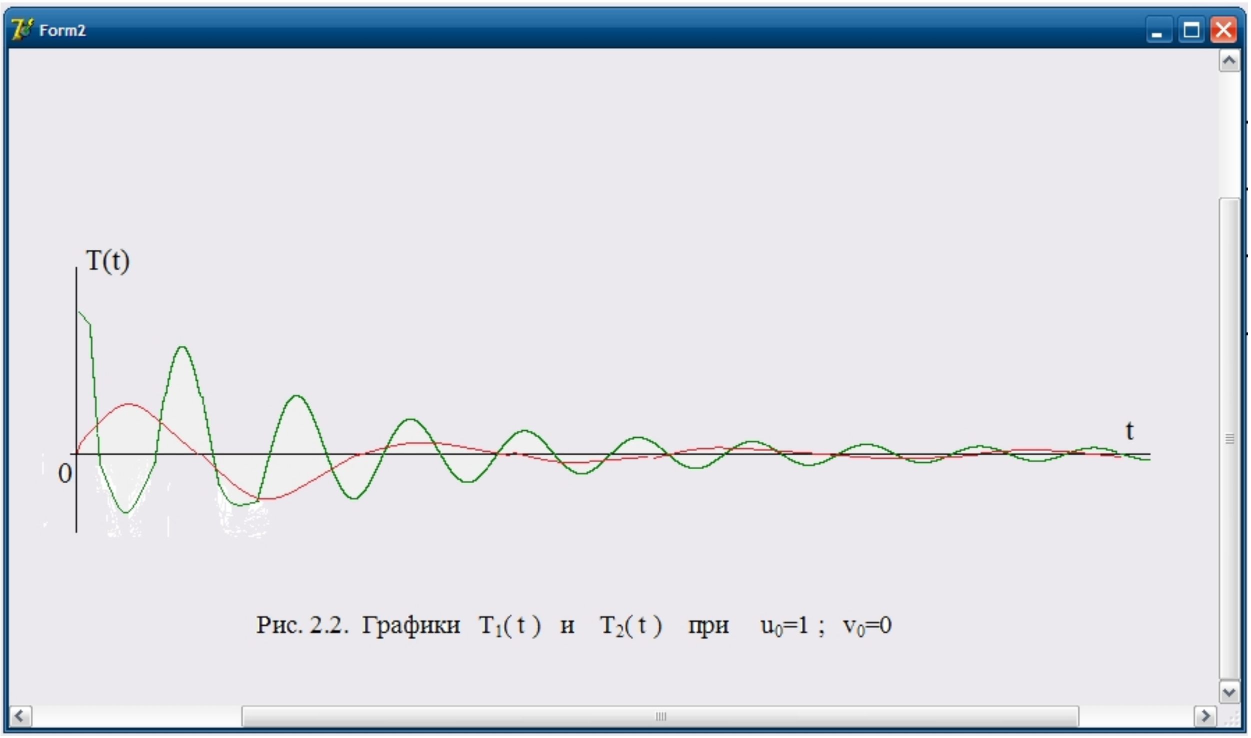

Fig. 2: Graphs of $T_1(t)$ and $T_2(t)$ at $u_0=1$; $v_0=0$

From the conducted computations it was obtained that taking into account further term of series improves the accuracy of the solution.

6. Discussion

The goal fot eh paper lies in the development of a generalized mathematical model and development of new methodology for solution of the integral-differential equation of viscoelastic systems with random rheology.

A mathematical method for solution was developed, which is suitable for both dynamic vicoelasticity problems and the problem of oscillation of viscoelastic systems.

With the help of Laplace integral transform, the integral-differential equation of oscillation of linear visoelastic systems for random kernels was solved.

A solution was achieved in the form of series and it was shown that at low frequencies of osscilation the influence of consequent terms of the series on the solution is insignificant and such influence increases with the increase of frequency.

The achieved solution was studied for Rzhanitsyn's kernel at different frequencies of a certain material.

Analysis of the achieved solution shows that taking into account consequent terms of the series improves the accuracy of the solution and that amplitudes of all terms of the series exponentially decrease in the course of time, and the phases are shifted.

The achieved results may be used directly for solving an array of applied problems in engineering computations of strength, durability and operational reliability of viscoelastic components of equipment.

The developed mathematical models and methods can be used in scientific research institutions, design organizations dealing with development and creation of new materials, in technical higher education establishments, as a new technique for solving linear and non-linear integral-differential equations with unknown integrands, which may be of interest for development of world science.

7. Conclusions

1. A general analytical solution method was proposed, which is which is suitable for both non-steady dynamic problems of viscoelasticity and the problems of oscillation of viscoelastic systems for random heritable functions describing mechanical properties of studied mediums.
2. Using the mentioned above method, a solution of integral-differential equation of linear viscoelastic systems' oscillation was developed in the form of series. It was shown that the first term of this series is the solution of integral-differential equation of viscoelastic systems' oscillation which was achieved using the known averaging method, and consequent memebrs provide the refinement of this solution.

With the use of numeric computations it was shown that at low frequencies the influence of consequent terms of the series is insignificant, and with the increase of the frequency such influence increases.

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